HW6 P71 solutions of applied DEs.

#1. $\frac{dh(t)}{dt} = -\frac{a}{A}\sqrt{kh(t)}$

(a) $C = \frac{a}{A}\sqrt{k} \Rightarrow \frac{dh}{dt} = -C\sqrt{h}$

$$\frac{dh}{\sqrt{h}} = -C\,dt$$

$$\int h^{-\frac{1}{2}}\,dh = \int -C\,dt$$

$$2h^{\frac{1}{2}} = -Ct + D$$

arbitrary constant. we use D b/c C is already used.

$$h = \begin{cases} \left(\frac{-Ct+D}{2}\right)^2 & \text{when } t \le \frac{D}{C} \\ 0 & \text{when } t > \frac{D}{C} \end{cases}$$

because $2h^{\frac{1}{2}} \ge 0$, so if $-Ct+D<0$, i.e. $t > \frac{D}{C}$.

(b) $\begin{cases} h(0) = 20 \\ h(20) = 0 \end{cases}$

$20 = \left(\frac{D}{2}\right)^2 \qquad D = 2\sqrt{20}$

$0 = \left(\frac{-20C+D}{2}\right)^2 \qquad C = \frac{D}{20} = \frac{2\sqrt{20}}{20}$

#3 (a) $R(t)$: # of rabbits

$\frac{dR}{dt} = 3\% \cdot R - 300 \Rightarrow \frac{dR}{dt} = 0.03R - 300.$

(b) t: time since 1991.

$R(0) = 12000$

$R(3) = ?$

$$\frac{dR}{dt} - 0.03R = -300$$

$a(t) = -0.03$

$m(x) = e^{\int a(t)dt} = e^{-0.03t}$

$$\frac{d}{dt}\left[e^{-0.03t}R\right] = -300e^{-0.03t}$$

$$e^{-0.03t}R = -300\int e^{-0.03t}\,dt$$

$$= -300\frac{e^{-0.03t}}{-0.03} + C$$

$$= 10000e^{-0.03t} + C$$

Plug in $R(0)=12000$, $12000 = 10000 + C \Rightarrow 2000 = C$

$\Rightarrow R = e^{0.03t}(10000e^{-0.03t} + 2000) = 10000 + 2000e^{0.03t}$

$R(3) = 10000 + 2000e^{0.09}$

#4 See this week's ws.